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Analysis I
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HW 2 [Revised 5 Sep]

Due: Sep 9 in class.

1. II.2.4
2. Let $g(x)$ be a bounded function in a neighborhood of a. Let $\lim_{x\to a} f(x) = 0$. Show that $\lim_{x\to a} f(x)g(x)$ exists and equals 0.
3. II.3.8
4. II.4.1
5. II.4.4
6. II.4.9. You may assume the result of II.4.7.
7. Prove that $f(x) = x^2$ is not uniformly continuous on $\mathbb{R}$.
8. [Revised] In this problem you will find examples of functions $f_n(x)$ defined on $(0, 1)$ such that

$$\lim_{x\to 0} \sum_{n=1}^{\infty} f_n(x) \neq \sum_{n=1}^{\infty} \lim_{x\to 0} f_n(x).$$

   (a) Find an example where the sum on the left hand side is $+\infty$ for all $x \in (0, 1)$.
   (b) Find an example where all the sums and limits are finite.
9. [Revised] *The track problem.* Here is a claim: if the temperature of a circular running track is given by a continuous function of a single position variable, there are two diametrically opposite points that have equal temperature.
   (a) Write the claim as a formal statement about continuous functions.
   (b) Prove that the claim is true or find a counterexample. Hint: Think about the intermediate value theorem.
10. If you were to present the Mean Value Theorem (Theorem 2.3) in class, write up the notes of what you would say. As an example, see my handwritten notes on the squeeze theorem from Day 4 of class. Follow the overall structure and comments in the document on presentations, available on the course website.