30 October 2014
Analysis I
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## Day 17 — Summary — Compactness

1. The function spaces $L^p$ can be defined as the completion of continuous functions under the $L^p$ norm. Later, we will define these spaces as the set of functions whose $p$th powers are Lebesgue integrable.

2. Definition: A subset $S$ of a normed vector space is (sequentially) compact if every sequence within the set has a subsequence that converges to an element of $S$.

3. A compact subset of a normed vector space is closed and bounded.

4. A closed subset of a compact set is compact.

5. A subset of $\mathbb{R}^k$ is compact if and only if it is closed and bounded.

6. Let $S$ be a compact subset of a normed vector space $V$, and let $f$ be a continuous function from $S$ to the normed vector space $F$. Then the image of $S$ under $f$ is compact.

7. A continuous function over a compact set in a normed vector space achieves its maximum and minimum.

8. A continuous function from a compact subset of a normed vector space to a normed vector space is uniformly continuous.

9. Consider a subset $S$ of a normed vector space. $S$ is sequentially compact if and only if any open cover of $S$ has a finite subcover. An alternative definition of compactness