21 April 2012
18.085
Computational Science and Engineering I
Paul E. Hand
hand@math.mit.edu

Problem Set 5

Due: **26 April 2012** in class.

Print or write out any Matlab input and output.

1. (10 points) The Heaviside function $H(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}$.
   (a) Plot $\int_{-\infty}^{x} \delta(y-2)dy$ as a function of x and express it in terms of Heaviside functions.
   (b) Same, but for $\int_{-\infty}^{x} \left(\delta(y-3) - \frac{1}{2}\delta(y+2)\right) dy$
2. (10 points) Without solving, sketch the solution to
$$-\frac{d^2u}{dx^2} = f(x)$$
$$u(0) = 0$$
$$\frac{du}{dx}(1) = 0$$
   for
   (a) $f(x) = -1$
   (b) $f(x) = \delta(x-1/2) + \delta(x-3/4)$
   (c) $f(x) = \begin{cases} 1 & \text{if } x < 1/2 \\ -1 & \text{if } x > 1/2 \end{cases}$
3. (10 points) Find the solution to
$$-\frac{d^2u}{dx^2} = \sin\frac{\pi x}{L}$$
$$u(0) = 0$$
$$u(L) = 0$$
4. (10 points) Find the solution to
$$-\frac{d^2u}{dx^2} = \delta(x-L/2)$$
$$u(0) = 0$$
$$\frac{du}{dx}(L) = 0$$

5. (20 points) Consider

$$\frac{d\mathbf{y}}{dt} = A\mathbf{y}$$
$$\mathbf{y}(0) = \mathbf{y}_0$$

where $\mathbf{y}(t) \in \mathbb{R}^2$ and A is a 2×2 matrix. Let $\mathbf{y}^n$ be the computed value of $\mathbf{y}$ at $t = n\Delta t$.

(a) Write out a Matlab expression to compute $\mathbf{y}^{n+1}$ in terms of $\mathbf{y}^n$, A, Δt, the identity matrix I, and the Matlab command "\" for

i. Forward Euler

ii. Backward Euler

iii. Trapezoidal rule

(b) Let $A = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$ and $\mathbf{y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. Use Matlab to compute $\mathbf{y}(1)$ using Forward Euler with $\Delta t = 0.01$

(c) Same as (b), but with Trapezoidal rule.