

Problem Set 2

Due: **1 March 2012** in class

1. (10 points) What is the rank of A ? Answer by stating a collection of columns that are independent, showing that those columns are independent, and arguing that there can be no additional independent columns.

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

2. (10 points) Find and sketch the null spaces of the following matrices.

(a) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

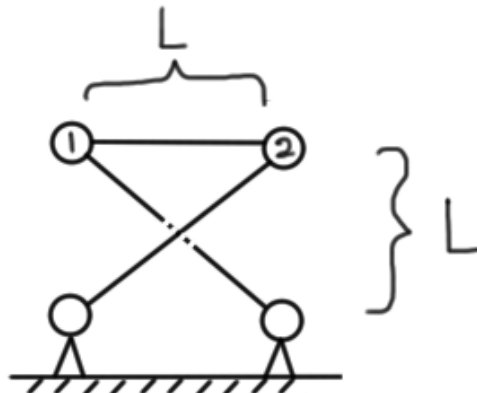
(b) $\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}$

3. (10 points) Find c_1, c_2, c_3, c_4, c_5 such that

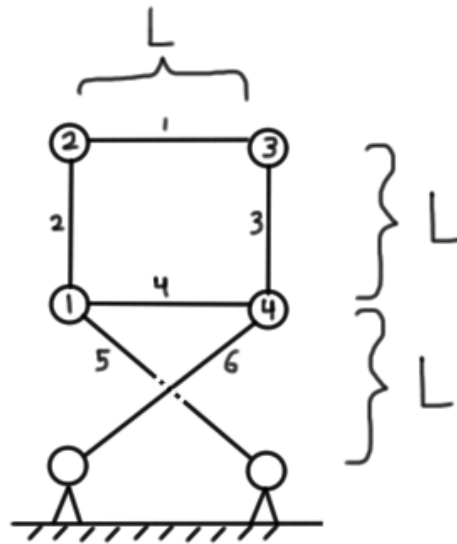
$$\begin{pmatrix} 6 \\ 4 \\ 5 \\ 6 \\ 9 \end{pmatrix} = c_1 \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + c_3 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + c_4 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} + c_5 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Let Matlab do the computation.

4. (20 points) Consider the following truss. The horizontal bar has length L . The diagonal bars have length $\sqrt{2}L$.



- (a) Write down the 3×4 linear system, $Ax = 0$, that enforces the constraint that all bars are of fixed length. How many independent modes of deformation of the truss are there?
- (b) Find the mode of deformation by physical reasoning. Check that it works by verifying $Ax = 0$.
- (c) Use Matlab to find the mode. Does it agree with (b)?
5. (20 points) Consider the following truss. All vertical and horizontal rods have length L . Both diagonal rods have length $\sqrt{2}L$.



- (a) Write out the 6×8 linear system, $Ax = 0$, that enforces the constraint that all bars are of fixed length. How many independent modes of deformation of the truss are there? You may assume A has rank 6.
- (b) Sketch the independent modes of deformation based on physical reasoning. Find one mode and check that it works by verifying $Ax = 0$.
- (c) Use Matlab to find the null space of A . Sketch the output. Does it include your answers from (b). Reconcile any discrepancies.